B–7–X

Roll No. .....

Total No. of Questions : 31]

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# 12th ARF(SZ)JKUT2024-25
# 307–X
# MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 80

## SECTION–A

## (OBJECTIVE TYPE QUESTIONS/ MULTIPLE CHOICE QUESTIONS)

1 each

**1.** If

$$\begin{vmatrix} x & 3 \\ 4 & -2 \end{vmatrix} = 0$$

then $x$ is equal to :

(A) 6

(B) –6

(C) 4

(D) –4

**2.** For a square matrix A, A(adj A) = ..... .



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**3.** Derivative of $e^{\log \tan x}$ w.r.t. $x$ is :

(A) $\cot x$

(B) $\tan x$

(C) $\sec^2 x$

(D) $\text{cosec}^2 x$

**4.** Second derivative of $\log (\log x)$ w.r.t. $x$ is :

(A) $\dfrac{1}{x \log x}$

(B) $\dfrac{1}{\log x}$

(C) $\dfrac{1}{x}$

(D) $\dfrac{-(1 + \log x)}{(x \log x)^2}$

**5.** The function :

$$f(x) = \frac{|x|}{x}, \quad x \neq 0$$

$$= 0, \quad x = 0$$

is discontinuous at $x = 0$. (True/False)

**6.** Maximum value of $\sin x$ is :

(A) 1

(B) –1

(C) 2

(D) –2



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7. $\int \tan x \, dx$ is equal to :

(A) log cos $x$ + C

(B) log sec $x$ + C

(C) log sin $x$ + C

(D) log cosec $x$ + C

8. Two lines are parallel if their direction ratios are ..... .

9. Projection of $\hat{i}$ on $\hat{j}$ is zero. (True/False)

10. Define optimal solution.

## SECTION–B

## (VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Show that the relation R in the set {1, 2, 3} given by

$$R = \{(1, 2), (2, 1)\}$$

is symmetric but neither reflexive nor transitive.

12. Prove that :

$$3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$$

$$x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$



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13. The length '$x$' of a rectangle is decreasing at the rate of 3 cm/minute and the width '$y$' is increasing at the rate of 2 cm/minute. When $x = 10$ cm and $y = 6$ cm, find the rate of change of the perimeter.

14. Find the unit vector in the direction of the vector

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

15. Find $|\vec{x}|$, if for a unit vector $\vec{a}$,

$$(\vec{x} - \vec{a}) . (\vec{x} + \vec{a}) = 12$$

16. Find :

$$\int \frac{\sin x}{1 + \cos x} dx$$

17. Evaluate :

$$\int_2^3 \frac{x\, dx}{x^2 + 1}$$



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18. A coin is tossed three times. If E is the event 'head on third toss' and F is the event 'heads on first two tosses'. Find P(E | F).

19. Let E and F be two events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent ?

20. Find the product :

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

## SECTION–C

**(SHORT ANSWER TYPE QUESTIONS)** 4 each

21. Find the relationship between $a$ and $b$ so that the function $f$ defined by

$$f(x) = ax + 1 \text{ if } n \leq 3$$

$$= bx + 3 \text{ if } x > 3$$

is continuous at $x = 3$.

22. Evaluate :

$$\int_0^1 x(1-x)^n \, dx$$

23. Differentiate $(\log x)^x + x^{\log x}$ w.r.t. $x$.



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**24.** Find the shortest distance between the lines :

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$$

and

$$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

**25.** If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that :

$$\vec{a} + \vec{b} + \vec{c} = 0$$

find the value of

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

**26.** Solve the linear programming problem graphically :

Maximize $z = 3x + 2y$

Subject to :

$$x + 2y \leq 10,$$

$$3x + y \leq 15,$$

$$x, y \geq 0.$$



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27. Let A and B be two sets. Show that :

$$f : A \times B \rightarrow B \times A$$

such that $f(a, b) = (b, a)$ is bijective function.

28. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ?

## SECTION–D

## (LONG ANSWER TYPE QUESTIONS) 6 each

29. For a matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

show $A^3 - 6A^2 + 5A + 11I = 0$.

*Or*

Solve the system of linear equations, using matrix method :

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$



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**30.** Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi \, d\phi$$

*Or*

Evaluate :

$$\int \frac{x \cos^{-1} x}{\sqrt{1 - x^2}} dx$$

**31.** Find the local maxima and local minima, if any, of the function :

$$f(x) = \sin x - \cos x, \ 0 < x < 2\pi.$$

*Or*

Find two positive numbers $x$ and $y$ such that $x + y = 60$ and $xy^3$ is maximum.



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